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Time series modeling of the interaction between deterministic and stochastic trends

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ABSTRACT

This study investigated a scenario where both the deterministic and stochastic trends coexist in a single realization. On exploring the monthly internally generated revenue of Akwa Ibom State in Nigeria from January, 2010 to December, 2014, we found that the deterministic trend with ARMA (1, 0) model adequately described the coexistence of both the deterministic and stochastic trends.

1. Introduction

Statistically, the trend corresponds to sustained and systematic variations over a long period of time, and is known to be associated with the structural causes of the phenomenon under study. In most socioeconomic time series, it often dominates other components of the series. Thus, its identification has always generated problems of conceptual complexity. This problem persists because the trend (as well as other remaining components) is a latent variable, and trends in time series clearly violate the conditions of stationarity (Qi and Zhang, 2008). Therefore, certain assumptions are required to handle the different types of nonstationarity processes. Actually, a given time series being a finite realization of a stochastic process, can always be represented by one of the components or a sum of several of them (Dagum and Luati, 2001; Moffat, 2007). On the different types of nonstationarity processes, Caiado and Crato (2005) considerd a deterministic linear trend process $X_t = a + b_t + \varepsilon_t$, where ε_t is a white noise term that can be transformed into a stationary process by subtracting the trend $a + b_t$, and a stochastic linear trend process such as the random walk model $(1- B) X_t = \varepsilon_t$ or $X_t = X_{t-1} + \varepsilon_t$. In particular, it is useful to distinguish between a random walk plus drift (unit root process) $X_t = \mu + X_{t-1} + \varepsilon_t$ and a deterministic trend in the form $X_t = a + \mu_t + \varepsilon_t$. In most cases, two of the simplest versions are (Eqs. 1 and 2):

Deterministic trend (DT):

$$X_t = b_t + \varepsilon_t \tag{1}$$

Stochastic trend (ST):

$$X_t = b_t + X_{t-1} + \varepsilon_t \tag{2}$$

where ε_t is a white noise with variance, $\sigma^2 = 1$, and $X_0 = 0$. Obviously (Eqs. 3 and 4),

$$E_{DS} (X_t) = E_{ST} (X_t) = b_t \tag{3}$$

but

$$V_{DS} (X_t) = 1 \text{ and } V_{ST} (X_t) = t. \tag{4}$$

Also, the expectation with respect to all information up to time t is zero.

As captured by Hamilton (1994), the stylized trend-cycle decomposition of time series (X_t) is as follows (Eq. 5):

$$
\begin{aligned}
X_t &= DT_t + Z_t \\
DT_t &= k + \delta t \\
Z_t &= \varphi_1 Z_{t-1} + \cdots + \varphi_p Z_{t-p} + \varepsilon_t, \ \varepsilon_t \sim WN(0, \sigma^2)
\end{aligned}
\tag{5}
$$

where DT_t is a deterministic trend and Z_t is an AR(p) process. If $|\varphi| < 1$, then Y_t is I(0) about the deterministic trend DT_t. If $= 1$, then $Z_t = \varphi_1 Z_{t-1} + \cdots + \varphi_p Z_{t-p} + \varepsilon_t = Z_0 + \sum_{j=1}^{t} \varepsilon_j$, a stochastic trend and X_t is I (1) with drift.

However, to avoid the complexity of the problem posed by a statistically vague definition, some researchers have resorted to two simple solutions: One consists of estimating the combined trend-cycle component; the other consists of defining the trend

in terms of the series length (Dagum and Dagum, 2006).

Generally, trend analysis in time series is of paramount interest to researchers since it provides information about the underlying properties of the series. Fuller (2009) opines that one of the reasons for estimating the trend function is to investigate the underlying properties of the time series. Wei (2006) identifies two classes of such properties to include the deterministic trend where the mean level of the process at time t is a pure deterministic function of time, and the stochastic trend where the mean level of the process changes through time randomly (Hamilton, 1994; Watson, 1986; Nelson and Plosser, 1982; Cryer and Chan, 2008). Although most macroeconomic time series are highly trended, it is difficult to determine whether the trend is deterministic or stochastic.

In our study, we consider a scenario where both the deterministic and stochastic trends coexist in a single realization.

2. Methodology

2.1. Augmented dickey-fuller test

Dickey and Fuller (1979) introduced Dickey-Fuller (DF) test statistic to examine the existence of unit root in a time series with an assumption that the underlying process follows the AR model. Also, Said and Dickey (1984) augment the basic autoregressive unit root test to accommodate the general ARMA (p, q) models and this test is called the Augmented Dickey-Fuller (ADF) test. Let the series be denoted by X_t using the regression (Eq. 6)

$$X_t = c_t + \beta X_{t-1} + \sum_{i=1}^{p-1} \varphi_i \Delta X_{t-i} + e_t \qquad (6)$$

where c_t is a deterministic function of the time index t and $\Delta X_j = X_j - X_{j-1}$ is the differenced series of X_t. Under the null hypothesis, X_t is I (1) which implies that $\beta = 1$. The ADF t-statistic based on least squares estimates of Eq. 6 is given by ADF-test $= \frac{\hat{\beta} - 1}{SE\ (\hat{\beta})}$.

3. Data analysis and discussion

Over the years, modeling internally generated revenue of a system has been of critical concern to all parties involved due to the variables of interest. In order to model the trend of the series, we examine the sequence plot in Fig. 1. From our observation, the plot appeared to contain a trend component.

Since our interest is in providing the justification for detecting between deterministic and stochastic trends, we carry out a statistical test to detect whether the trend component is deterministic or stochastic or both, using Augmented Dickey – Fuller test (ADF). We apply the ADF to the regression model that contains both intercept and trend component. The trend is said to be deterministic if the series is stationary with trend, otherwise it is stochastic. According to Table 1, the ADF test

statistic value is -5.4546 and the test critical values are -4.1213, -3.4878, and -3.1723 at 1%, 5% and 10% levels of significance, respectively. Since the ADF test statistic is greater than all the critical values in absolute value, the null hypothesis (X_t is nonstationary and the trend component is not significant) is rejected with the decision that the series is stationary with a deterministic trend.
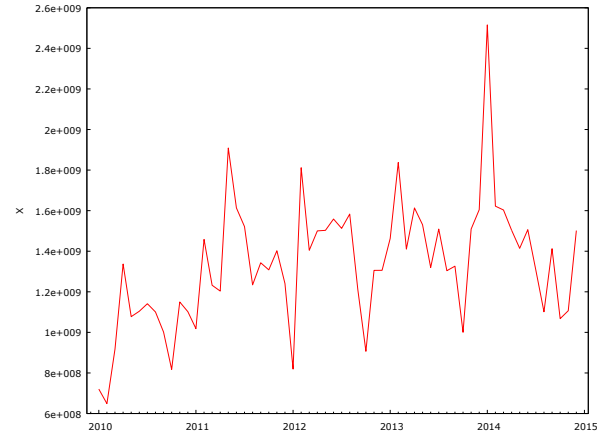

Fig. 1: Sequence plot of internally generated revenue

Table 1: Augmented dickey-fuller test

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-5.454627	0.0002
Test critical values:	1% level	-4.121303	
	5% level	-3.487845	
	10% level	-3.172314	

Null Hypothesis: X has a unit root; Exogenous: Constant, Linear Trend; Lag Length: 0 (Automatic - based on SIC, maxlag=10)
*MacKinnon (1996) one-sided p-values.

3.1. Deterministic trend model

The fitted trend model is presented in Eq. 7 (Excerpts from Table 2):

$$X_t = 1.1108e + 09 \quad + \quad 7.3613e + 06$$

Trend

s.e	$7.4399e + 07$	$2.1212e + 06$
t-ratio	14.9296	3.4703
p-value	< 0.0001	0.0010

(7)

From the deterministic trend model in Eq. 7, it is observed that the time trend in the model is significant since the p- value = 0.0010 < 0.05 level of significance. But the diagnostic checking of the deterministic trend model indicates that the model is not adequate since the ACF of the residuals of the deterministic trend model (Fig. 2) is auto correlated which is a clear indication of randomness in X_t. The fact that randomness exists in the residuals of the deterministic model shows that X_t also exhibits a stochastic trend. Hence, the deterministic trend model needs to be refined to contain an ARMA process.

Again, we examine the ACF and the PACF of the residuals of deterministic trend model (Fig. 2), and

they indicate tentatively that deterministic trend with ARMA (1, 0), ARMA (0, 1) and ARMA (1, 1) models respectively could be appropriate.

Comparing their information criteria as appeared in Tables 3, 4, and 5, we observed that deterministic trend with ARMA (1, 0) model has the smallest information criteria.

Table 2: Output of deterministic trend

	Coefficient	Std. Error	t-ratio	p-value	
Const.	1.11075e+09	7.4399e+07	14.9296	<0.0001	***
Time	7.36129e+06	2.12122e+06	3.4703	0.0010	***
Mean dependent var	1.34e+09		S.D. dependent var	3.10e+08	
Sum squared resid	4.70e+18		S.E. of regression	2.85e+08	
R-squared	0.171938		Adjusted R-squared	0.157661	
F(1, 58)	12.04310		P-value(F)	0.000988	
Log-likelihood	−1252.105		Akaike criterion	2508.209	
Schwarz criterion	2512.398		Hannan-Quinn	2509.848	
Rho	0.324102		Durbin-Watson	1.317962	

Model 1: OLS, using observations 2010:01-2014:12 (T = 60). Dependent variable: X
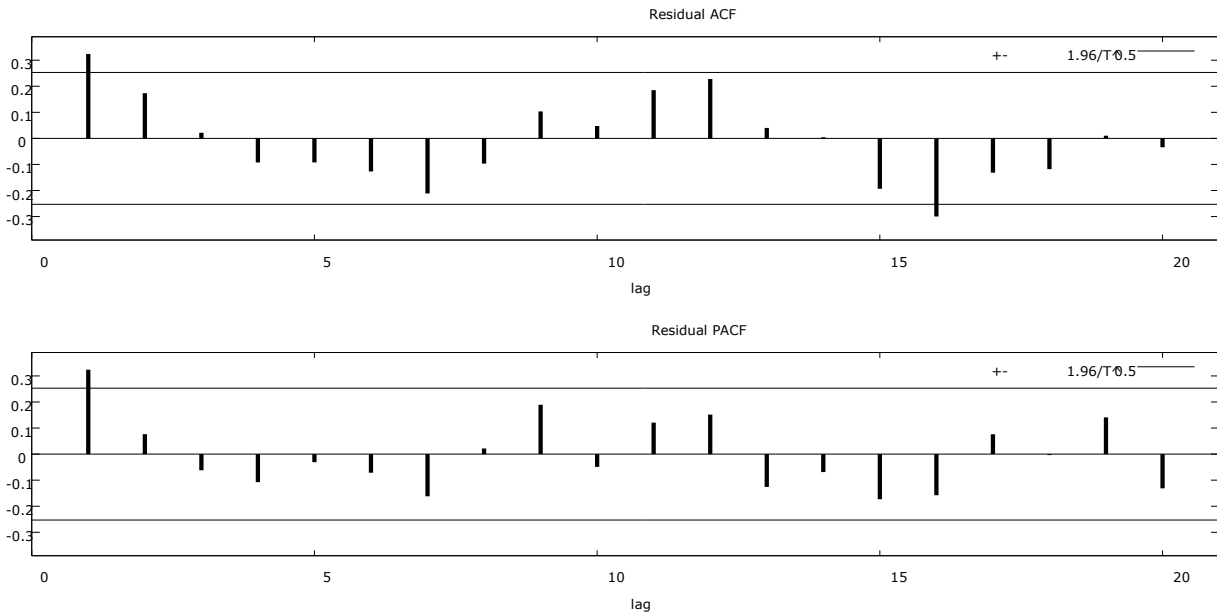

Fig. 2: ACF and PACF of the residuals from deterministic trend model

Therefore, our chosen model is the deterministic trend with ARMA (1, 0) model. The estimated model is presented in Eq. 8 (Excerpts from Table 3):

$$X_t = 1.1062e+09 + 7.6421e+06\,Trend + 0.3304X_{t-1}$$

s.e	9.8791e+07	2.8738e+06	0.1228
z – ratio	11.1974	2.6592	2.6897
p – value	< 0.0001	0.0078	0.0072

(8)

From Eq. 8, it is observed that both trend and the ARMA (1, 0) processes are significant since their respective p – values are less than 0.05 level of significance.

The diagnostic checking of the model in Eq. 8 reveals that the residuals are no more auto correlated (Fig. 3). The implication is that both deterministic and stochastic trend coexist in the time series X_t.

Table 3: Output of deterministic trend with ARMA (1, 0) model

	Coefficient	Std. Error	Z	p-value	
Const	1.1062e+09	9.87911e+07	11.1974	<0.0001	***
phi_1	0.330366	0.122826	2.6897	0.0072	***
TREND	7.64207e+06	2.87381e+06	2.6592	0.0078	***
Mean dependent var	1.34e+09		S.D. dependent var	3.10e+08	
Mean of innovations	2123045		S.D. of innovations	2.64e+08	
Log-likelihood	−1248.706		Akaike criterion	2505.411	
Schwarz criterion	2513.789		Hannan-Quinn	2508.688	

Model 1: ARMAX, using observations 2010:01-2014:12 (T = 60); Dependent variable: X. Standard errors based on Hessian

4. Conclusion

According to Rao (2010), deterministic trends are useful in unit root tests and in the estimation of the models with cointegration techniques. The implication of allowing for such trends is that if the model is shocked, after some departures from the trend, the variables will return to their trend values.

On the other hand, models with stochastic trend (structural time series models) are useful in some instances. Firstly, it may be hard to identify multiple structural breaks in the deterministic trend when the sample size is small. Secondly, implementing endogenous multiple structural break tests remains a demanding exercise. Thirdly, standard classical methods of estimation can be used to estimate the effects of additional explanatory variables in structural time series models. Finally, Harvey (1985) points out that the stochastic and deterministic trend hypotheses are nested within the structural time series approach and can be evaluated with estimated values of the hyper parameters, although the power of this test is not known.

Table 4: Output of deterministic trend with ARMA (0, 1) model

	Model 3: ARMAX, using observations 2010:01-2014:12 (T = 60)				
	Dependent variable: X				
	Standard errors based on Hessian				
	Coefficient	Std. Error	Z	p-value	
Const	1.1128e+09	8.55138e+07	13.0131	<0.0001	***
theta_1	0.264255	0.109452	2.4144	0.0158	**
TREND	7.50316e+06	2.49545e+06	3.0067	0.0026	***
Mean dependent var	1.34e+09		S.D. dependent var		3.10e+08
Mean of innovations	901967.2		S.D. of innovations		2.67e+08
Log-likelihood	−1249.444		Akaike criterion		2506.887
Schwarz criterion	2515.264		Hannan-Quinn		2510.164

Table 5: Output of deterministic trend with ARMA (1, 1) model

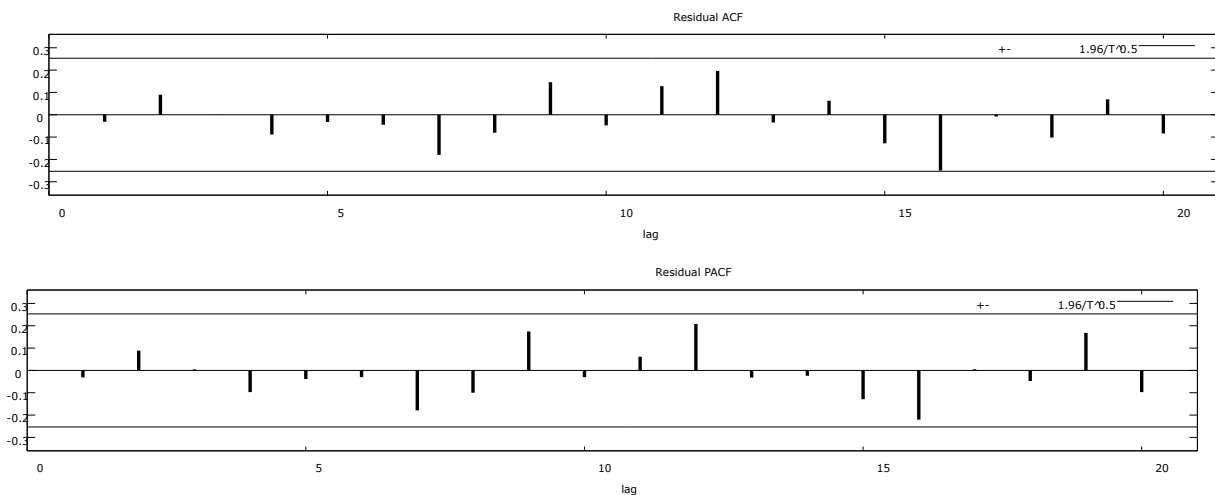|	Model 2: ARMAX, using observations 2010:01-2014:12 (T = 60)				
	Dependent variable: X				
	Standard errors based on Hessian				
	Coefficient	Std. Error	Z	p-value	
Const	1.10134e+09	1.06576e+08	10.3339	<0.0001	***
phi_1	0.496663	0.268404	1.8504	0.0643	*
theta_1	−0.181411	0.286809	−0.6325	0.5271	
TREND	7.71014e+06	3.08148e+06	2.5021	0.0123	**
Mean dependent var	1.34e+09		S.D. dependent var		3.10e+08
Mean of innovations	3182619		S.D. of innovations		2.63e+08
Log-likelihood	−1248.531		Akaike criterion		2507.062
Schwarz criterion	2517.534		Hannan-Quinn		2511.158
Log likelihood	−1248.531		Akaike criterion		2507.062
Schwarz criterion	2517.534		Hannan Quinn		2511.158

Fig. 3: ACF and PACF of the residuals from deterministic trend with ARMA (1, 0) model

If the variances of the disturbances of the level and slope of trend are zero (known as hyper parameters), the structural time series model implies that a deterministic trend is preferable to a stochastic trend. Thus, in the light of these observations, it is hard to conclude on the approach that is better and our view is that both methods are worth using, especially to keep up with further research works in both approaches. However, in time series, it is hard to ascertain whether a single realization contains a trend and to determine whether the trend is deterministic or stochastic.

Also, distinguishing between deterministic and stochastic trends always seems contentious but our study considers a scenario where both the deterministic and stochastic trends coexist in a single realization. To determine whether a trend exists in X_t, we applied the Augmented Dickey-Fuller test and the result showed that X_t is stationary with trend. Hence, the deterministic trend is said to exist in X_t. We progressed to fit a deterministic trend model to X_t. Meanwhile, the diagnostic checking indicated the presence of randomness in the residuals of the fitted deterministic trend model

which is a clear sign that stochastic trend equally exist in X_t and such randomness could be modeled by an ARMA (1, 0) process. In all, deterministic trend with ARMA (1, 0) model was found to be adequate in modeling both the deterministic and stochastic trends in the series.

Moreover, this study could be extended to include the need to model separately the deterministic trend and stochastic trend in X_t in order to check which one will perform better.

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